Grammars and languages

Patryk Kiepas

March 21, 2017

1 Grammar class and generated language

Assign to a grammar $G = (V_T, V_N, P, S)$ corresponding **Chomsky's class** (*unrestricted, context-sensitive, context-free, regular*) and **explain your choice**. Then **describe the language** $L(G)$ generated by this grammar G. Only a set of rule P is given. S is always a *start rule*. V_T and V_N are derived from the set of rules P.

1.

$$S \rightarrow aS \mid bS \mid aA$$
$$A \rightarrow aA \mid bA \mid aB$$
$$B \rightarrow aB \mid bB \mid \epsilon$$

Answers:

- Grammar class: *regular*
- Language L consists of words over alphabet $\{a, b\}$ with at least two a:
  $L = \{(a^*b^*)^*\mathbf{a}(a^*b^*)^*\mathbf{a}(a^*b^*)^*\}$ or
  $L = \{\{a, b\}^*\mathbf{a}\{a, b\}^*\mathbf{a}\{a, b\}^*\}$ or
  $L = \{.^*\mathbf{a}.^*\mathbf{a}.^*\}$, where . means any terminal from the alphabet $\{a, b\}$.
  These three notations are equivalent.

2.

$$S \rightarrow aS \mid bS \mid aA$$
$$A \rightarrow aB$$
$$B \rightarrow aB \mid bB \mid \epsilon$$

Answers:

- Grammar class: *regular*
- Language L consists of words over alphabet $\{a, b\}$ with at least two consequent a:
  $L = \{(a^*b^*)^*\mathbf{aa}(a^*b^*)^*\}$ or
  $L = \{\{a, b\}^*\mathbf{aa}\{a, b\}^*\}$ or
  $L = \{.^*\mathbf{aa}.^*\}$, where . means any terminal from the alphabet $\{a, b\}$.
  These three notations are equivalent.

3.

$$S \to ab \mid LSQ$$
$$La \to aaL$$
$$bQ \to Qbb$$
$$LLQQ \to LQ$$
$$aLQb \to ab$$

Answers:

- Grammar class: *unrestricted*
- Language L consists of words over alphabet $\{a, b\}$ of repeated a followed by the same number of repeated b: $1, 2, 4, 8, \ldots$: $L = \{a^{\mathbf{m}}b^{\mathbf{m}} \mid \mathbf{m} = 2^n, n \geq 0\}$

4.

$$S \to ADBC$$
$$D \to DD$$
$$DB \to BEEE$$
$$ABE \to aAB$$
$$aABC \to a$$

Answers:

- Grammar class: *unrestricted*
- Language L consists of words over alphabet $\{a, b\}$ of a repeated with multiplicity of 3: $L = \{a^m \mid m = \mathbf{3n}, n \geq 1\}$.

5.

$$S \to aSBC \mid abC$$
$$CB \to BC$$
$$bB \to bb$$
$$bC \to bc$$
$$cC \to cc$$

Answers:

- Grammar class: *context-sensitive*
- Language L consists of words over alphabet $\{a, b, c\}$ of repeated n-times a, then n-times b followed by n-times c: $L = \{a^{\mathbf{n}}b^{\mathbf{n}}c^{\mathbf{n}} \mid \mathbf{n} \geq 1\}$

6.

$$S \to abC \mid Ac \mid Dbc \mid aEF \mid aB$$
$$A \to ab$$
$$Db \to ab$$
$$B \to bc$$
$$bC \to bc$$
$$F \to c$$
$$Ec \to bc$$

Answers:

- Grammar class: *context-sensitive*
- Language L consists of one word abc: $L = \{abc\}$

2 Grammar creation

Given a description of a language L create grammar $G = (V_T, V_N, P, S)$ that generates this language $L(G)$. List a set of production rules P.

1. $L = \{ab, bbc, ccca, aaaab, bbbbbc, ccccca, aaaaaaab, ...\}$

    Answers:

    - Grammar class: *context-free*

    $$S \to A \mid B \mid C$$
    $$A \to ab \mid aaaA$$
    $$B \to bbc \mid bbbB$$
    $$C \to ccca \mid cccC$$

2. $L = \{a, b, ab, ba, aba, bab, abab, baba, ababa, babab, ...\}$

    Answers:

    - Grammar class: *context-free*

    $$S \to A \mid B$$
    $$A \to a \mid ab \mid abA$$
    $$B \to b \mid ba \mid baB$$

3

3. $L = \{(aabb)^i | i \geq 1\}$

   Answers:
   - Grammar class: *context-free*

   $$S \rightarrow aabbS \mid \epsilon$$

4. $L = \{a^i c b^i | i \geq 0\}$

   Answers:
   - Grammar class: *context-free*

   $$S \rightarrow aSb \mid c$$

5. $L = \{(), (()), ()(), ()(()()), (()(()))(), ...\}$ – language of correctly nested parentheses:

   Answers:
   - Grammar class: *context-free*

   $$S \rightarrow SS \mid (S) \mid \epsilon$$