

Grammars and languages (II)

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1 Grammar class and generated language

Assign to a grammar $G = (V_T, V_N, P, S)$ corresponding **Chomsky's class** (*unrestricted, context-sensitive, context-free, regular*) and **explain your choice**. Then **describe the language** $L(G)$ generated by this grammar G . Only a set of rule P is given. S is always a *start rule*. V_T and V_N are derived from the set of rules P .

1.

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow baA \mid aA \mid ba \mid a \end{aligned}$$

Answers:

- Grammar class: *context-free*
- Language L consists of words over alphabet $\{a, b\}$ with a at the begin and end. In the middle there are a and b mixed up, but every b is separated by a from both sides:
 $L = \{\mathbf{a}(a^*(ab)^*(ba)\mathbf{a})\}$ or
 $L = \{\mathbf{a}\{a, ab\}^*(ba)\mathbf{a}\}$
These two notations are equivalent.

2.

$$S \rightarrow aS \mid aSb \mid \epsilon$$

Answers:

- Grammar class: *context-free*
- Language L consists of words over alphabet $\{a, b\}$ with a repeated n -times followed by b repeated m -times, where $n \geq m$:
 $L = \{a^n b^m \mid n \geq 0, m \geq 0, n \geq m\}$

3.

$$S \rightarrow SS \mid aSb \mid bSa \mid ab \mid ba$$

Answers:

- Grammar class: *context-free*
- Language L consists of words over alphabet $\{a, b\}$ with a and b mixed up freely, but with the same number of a and b :
 $L = \{\{a, b\}^* \mid \text{numOfOccurences}(a) == \text{numOfOccurences}(b)\}$

2 Grammar creation

Given a description of a language L create grammar $G = (V_T, V_N, P, S)$ that generates this language $L(G)$. List a set of production rules P .

1. $L = \{abb, abbaab, abbaababb, abbaababbaab, \dots\}$

Answers:

- Grammar class: *context-free*

$$\begin{aligned} S &\rightarrow abb \mid abbA \\ A &\rightarrow aab \mid aabS \end{aligned}$$

2. $L = \{ab^n c^{2n+m} a \mid n \geq 1, m \geq 1\}$

Answers:

- Grammar class: *context-free*

$$\begin{aligned} S &\rightarrow aAa \\ A &\rightarrow bAcc \mid bBcc \\ B &\rightarrow cB \mid c \end{aligned}$$

3. $L = \{ba^i cb^{i+1} \mid i \geq 1\}$

Answers:

- Grammar class: *context-free*

$$\begin{aligned} S &\rightarrow bAb \\ A &\rightarrow aAb \mid acb \end{aligned}$$