Introduction to theory of languages

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Course information

Plan

- Saturday, 25th of February 2017 lecture (introduction to grammars)
- Saturday, 4th of March 2017 lecture (grammars and ANTLR)
- Saturday, 11th of March 2017 exercises (grammars)
- Saturday, 25th of March 2017 exercises (ANTLR)
- S Exam

Any questions?

Ask by mail: kiepas@agh.edu.pl

Course web-page

 $\label{eq:http://home.agh.edu.pl/~kiepas} \rightarrow \textbf{Teaching} \rightarrow \textbf{Introduction to} \\ \textbf{theory of languages (2017)}$

Outline

Course information

Theory 2

- Languages
- Grammars
- Hierarchy of grammars
- Examples of grammars



Practice

- Notations of grammar
- ANTLR
- ANTLR examples

Introduction

Linguistics

Scientific study of languages. Involves analysis of language:

- form language evolution and task
- context environment of language usage
- semantics the meaning of the language

Some important aspects

- Phonetics
- Articulation
- Perception
- Acoustic features
- Morphology
- Syntax

Language types

Natural languages

- Ordinary evolves naturally in humans without planning
- Controlled a restricted subset of natural language in order reduce or eliminate ambiguity and complexity

Artificial languages

- Constructed (planned a priori or a posteriori)
 - Engineered languages experiments in logic, philosophy, linguistics
 - Auxiliary languages international communication (e.g. Esperanto, Ido, Interlingua)
 - Artistic languages aesthetic pleasure or humorous effect (e.g. Klingon)
- Formal
 - Computer programming languages (e.g. Java, Haskell, C, C++, Ruby)
 - Files and formats descriptions (e.g. YAML, JSON, XML)

Description of natural languages

A really small bit of history

- In the late 1950's Noam Chomsky tried to describe natural languages
- Important paper: "Three models for the description of language", Noam Chomsky (1956).
- In a result of his research two disciplines originated:
 - **1** Theory of formal grammars
 - Q Generative (transformational) grammars



Figure 1: Professor of Linguistics (Emeritus) at MIT, Cambridge

What we know now?

- Description of natural languages is hard
- Description of any natural languages might be impossible

Why this is important?

- Better understanding of language creation processes
- More insights into functioning of our brain
- Natural language processing (NLP)
 - Translations (e.g. Google Translator)
 - Synthesis (e.g. speech generation)
 - Perceiving (e.g. robots, voice-control)

Description of formal languages

Result

Description of natural languages help us describe an artificial (formal) ones

Programming languages

- Protocol for communication with the computer
- Performing operations and computations
- Interpretation and execution
- Compilation
- Static code analysis

Data formats

- Structured data
- Interchangeable model for communication and data transmission

Alphabet

Alphabet

A set $\boldsymbol{\Sigma}$ of available symbols, the simplest elements in the language

Examples

- binary alphabet $\{0,1\}$
- decimal numbers $\{0, 1, 2, 3, ..., 9\}$
- Latin alphabet $\{a, b, c, d, ..., z\}$
- Cyrillic

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[1]	[k]	[i]	[h]	[z]	[f]	[e]	[d]	[k]	[b]	[a]
	Х	Y	Τ	ζ	٥	Q	1	0	۲	ግ
		Y บ			-	•	•	-		•

Figure 2: Ancient Latin alphabet

Word (I)

Word

Word *w* is a sequence of *N* symbols $w = x_1x_2...x_N$ where $x_i \in \Sigma$ (e.g. 010110, *ABCDAAE*)

Length

Length of the word w is a number of symbols it contains |w| = N(e.g. |010110| = 6, |ABCDAAE| = 7)

Empty word

Special word ϵ with length $|\epsilon| = 0$

Word (II)

Words examples

- w = 010110 word over alphabet $\Sigma = \{0, 1\}$
- w = abc13dj3 word over alphabet $\Sigma = \{a, b, ...z, 0, 1, ...9\}$
- w = ACGTCCGGTA word over alphabet $\Sigma = \{A, C, G, T\}$

Kleene star (closures)

•
$$\Sigma^+$$
 – set of all nonempty words $\Sigma^+ = \Sigma^* ackslash \{\epsilon\}$

Closures examples

• if
$$\Sigma = \{a\}$$
 then $\Sigma^* = \{\epsilon, a, aa, aaa, aaaa, aaaaa, aaaaaa, ...\}$

• if
$$\Sigma = \{a, b\}$$
 then $\Sigma^+ = \{a, b, aa, bb, ab, ba, aaa, bbb, ...\}$

• if
$$\Sigma = \{a, b, ..., z\}$$
 then $\Sigma^+ = \{cat, dog, a, aa, aaa, ...\}$

Language

Definition

Formal language $L\subseteq \Sigma^*$ is a subset of all words built over an alphabet Σ

Examples

- Language L_1 of palindromes in English $L_1 = \{mum, hannah, madam, ...\}$
- \bullet Morse code with alphabet $\Sigma=\{\cdot,-\},$ $L_2=\{\cdot-,-\cdot\cdot...,-\cdot\cdot\}$
- Empty language
- English language
- Language L_3 with the set of words with fixed-size of N
- Language $L_4 = \{a^n b^n | n \ge 1\}$
- Language $L_5 = \{abc^n de | n \ge 0\}$
- Language $L_6 = \{a^m | m = 3n \land n \ge 1\}$

Grammar

Grammar

- Description of a language and its syntax
- A recipe for composing elements of an alphabet into sentence

Definition

Grammar is a system $G = (V_T, V_N, P, S)$ where:

- V_T terminals (alphabet Σ)
- V_N nonterminals
- $V_N \cap V_T = \emptyset$
- $V = V_N \cup V_T$ vocabulary
- P production rules ($P \subseteq V^+ \times V^*$)
- V_N, V_T, P are finite, nonempty sets
- S start symbol, $S \in V_N$

Grammar and languages

Derivation

- Let $\alpha, \beta \in V$, then we say that:
 - β derives directly from α (i.e. α ⇒ β) if there exists production rule p ∈ P that obtains β from α
 - α_n derives from α₁ (i.e. α₁ ^{*}⇒ α_n) if there exists a sequence of direct derivations giving in the result α_n :
 α₁ ^{p₁}⇒ α₂ ^{p₂}⇒ α₃ ^{p₃} ... ^{p_n}⇒ α_n, where {p_i : 0 ≤ i ≤ k ∧ p_i ∈ P}

Grammars and languages

- Sentence $w \in V_T^*$ is generated by grammar G if w derives from S $(S \stackrel{*}{\Rightarrow} w)$
- We say grammar G generate language L(G) if it contains all sentences derived from S, in a result L(G) = {w ∈ V_T^{*} : ∃(S ^{*}⇒ w)}
- Two grammars G_1 and G_2 have (weak) equivalence if $L(G_1) = L(G_2)$

Grammar examples

Grammar

Let
$$G = (V_N, V_T, P, S)$$
, where
• $V_N = \{S\}$

•
$$V_T = \{a, b\}$$

•
$$P = \{ S \rightarrow aS, S \rightarrow b \}$$

Derivations

$$\mathsf{S} \Longrightarrow \mathsf{aS} \Longrightarrow \mathsf{aaS} \Longrightarrow \mathsf{aaaS} \Longrightarrow \mathsf{aaaaS} \Longrightarrow \mathsf{aaaaaS} \Longrightarrow \mathsf{aaaaaS} \Longrightarrow \mathsf{aa}$$

Language

$$L(G) = \{a^n b\}, \text{ where } n \ge 0$$

Example sentences

b, ab, aab, aaab, aaaab, aaaaab, aaaaaab, aaaaaaab, aaaaaaab, ...

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Chomsky's hierarchy

Hierarchy

- Describe the grammar expressiveness
- Observation: regular language is also a context-free grammar etc. (Regular ⊆ Context-free ⊆ Context-sensitive ⊆ Unrestricted)
- The nested grammar is more restrictive than the enclosing one

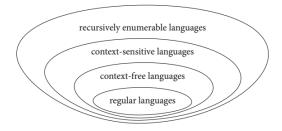


Figure 3: Chomsky's hierarchy

Limiting conditions

For all production rules $\forall (\alpha \rightarrow \beta) \in P$ it is true:

First condition

• $|lpha| \leq |eta|$ - they don't decrease length of a word, where $lpha, eta \in V^*$

Second condition

- $\alpha \in V_N$ is a nonterminal
- $\beta \in V^+$ is not empty

Third condition

- $\alpha \in V_N$ is a nonterminal
- β has a form $\beta = a$ or $\beta = aB$ where $a \in V_T, B \in V_N$

Type-0 : Recursively enumerable (unrestricted) grammar

Description

Type-0 grammar has no limitations (is unrestricted)

Valid production rules

Production rules have form of $\alpha \rightarrow \beta$, where $\alpha, \beta \in V$

- $aaaA \rightarrow aBb$
- $LLQQ \rightarrow LQ$
- $S \to \epsilon$
- $C \rightarrow cC$
- $D \rightarrow E$

• abcD
ightarrow abc

Type-0 : Grammar example

Grammar

- Let $G = (V_N, V_T, P, S)$, where
 - $V_N = \{S, A, B, C, D\}$

•
$$V_T = \{a\}$$

• $P = \{S \rightarrow ADBC, D \rightarrow DD, DB \rightarrow BEEE, ABE \rightarrow aAB, aABC \rightarrow a\}$

Derivations

$$S \implies ADBC \implies ADBC \implies ABEEEC \implies aABEEC \implies aABEC \implies aaABEC \implies aaaABC \implies aaa$$

Language

$$L(G) = \{a^m\}$$
, where $m = 3n \wedge n \geq 1$

Example sentences

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Type-1 : Context-sensitive grammar

Description

Productions rules of *type-1 grammar* don't decrease the length of the words (i.e. $|\alpha| \le |\beta|$) during derivations

Valid rules

- $S \to \epsilon$
- $C \rightarrow cC$
- $D \rightarrow E$
- $\bullet \ aBc \to abBc$

Invalid rules

- aaaA
 ightarrow aBb
- $LLQQ \rightarrow LQ$
- $abcD \rightarrow abc$

Type-1 : Grammar example

Grammar

Let $G = (V_N, V_T, P, S)$, where

•
$$V_N = \{S, A, B, C, D, E, F\}$$

•
$$V_T = \{a, b, c\}$$

•
$$P = \{S \rightarrow abC | Ac | Dbc | aEF | aB, A \rightarrow ab, Db \rightarrow ab, B \rightarrow bc, bC \rightarrow bc, F \rightarrow c, Ec \rightarrow bc\}$$

Derivations

•
$$S \implies abC \implies abc$$

•
$$\mathsf{S} \Longrightarrow \mathsf{Ac} \Longrightarrow \mathsf{abc}$$

•
$$S \implies aEF \implies aEc \implies abc$$

Language and example sentence

 $L(G) = \{abc\}$

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Type-2 : Context-free grammar

Description

Rules $A \to \beta$ in context-free grammar have one variable (nonterminal) on the left hand side $(A \in V_N)$ and they derive into any word $(\beta \in V^*)$

Valid rules	
• $S \to \epsilon$	
• $C \rightarrow cC$	
• $D \rightarrow E$	
• $F ightarrow abcdef$	
Invalid rules	
invaliu rules	
• aaa $A ightarrow aBb$	
• $LLQQ \rightarrow LQ$	
• $aBc ightarrow abefBc$	

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Type-2 : Grammar example

Grammar

Let
$$G = (V_N, V_T, P, S)$$
, where

•
$$V_N = \{S, A, B\}$$

•
$$V_T = \{a, b\}$$

•
$$P = \{S \rightarrow aSB, S \rightarrow A, A \rightarrow ab, B \rightarrow b\}$$

Derivations

$$S \implies aSB \implies aaSBB \implies aaSBb \implies aaSbb \implies aaabbb$$

Language

$$L(G) = \{a^n b^n\}$$
, where $n \ge 1$

Example sentences

 $ab, aabb, aaabbb, aaaabbbbb, aaaaabbbbbb, aaaaaabbbbbbb, \ldots$

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Type-3 : Regular grammar

Description

Rules in regular grammar have form of $A \rightarrow a$ and $A \rightarrow aB$ (right recursion) or $A \rightarrow Ba$ (left recursion), where $A, B \in V_N$ and $a \in V_T$

Valid rules	
• $C \rightarrow cD$	
• $D \rightarrow Dc$	
• $S \rightarrow b$	

Invalid rules

- $S \to \epsilon$
- $D \rightarrow E$
- aBc
 ightarrow abefBc
- $F \rightarrow abcdef$

Grammar example – regular

Grammar

Let
$$G = (V_N, V_T, P, S)$$
, where

•
$$V_N = \{S, B\}$$

•
$$V_T = \{a, b\}$$

•
$$P = \{S \rightarrow aB, B \rightarrow bS, B \rightarrow b\}$$

Derivation

$$S \implies aB \implies abS \implies abaB \implies ababS \implies ababaB \implies \dots$$

Language

$$L(G) = \{(ab)^n\}, \text{ where } n \geq 1.$$

Example sentences

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Normal forms

Let $A, B \in V_N$, $a \in V_T$ and $\beta \in V_N^*$, then:

The Chomsky normal-form

All productions of a grammar have the form of A
ightarrow BC or A
ightarrow a

The Greibach normal-form

All productions of a grammar have the form A
ightarrow aeta

Theorem

Any context-free language can be generated by a grammar in **Chomsky** normal-form

Theorem

Any context-free language can be generated by a grammar in **Greibach** normal-form

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Normal forms

We saw that $L(G) = \{a^n b^n\}$, where $n \ge 1$ is a language created by a grammar G with production rules:

•
$$P = \{S \rightarrow aSB, S \rightarrow A, A \rightarrow ab, B \rightarrow b\}$$

But there are other rules P' from grammar G' in a Chomsky normal-form that will give us the same language L(G) = L(G'):

•
$$P' = \{S \rightarrow CB, D \rightarrow SB, S \rightarrow CD, C \rightarrow a, B \rightarrow b\}$$

Derivation a^3b^3

$$S \implies CD \implies aD \implies aSB \implies aCDb \implies aaDb \implies$$

And rules P'' from grammar G'' in a Greibach normal form:

•
$$P'' = \{S \rightarrow aD, D \rightarrow aDB, D \rightarrow aBB, B \rightarrow b\}$$

Derivation
$$a^3b^3$$

 $S \implies aD \implies aaDB \implies aaaBBB \implies aaabBB \implies aaabbB \implies aaabbb$
Patrik Kiense (MINES ParisTech & ACH) Theory of languages March 11, 2017 – 27, 45

Derivation trees

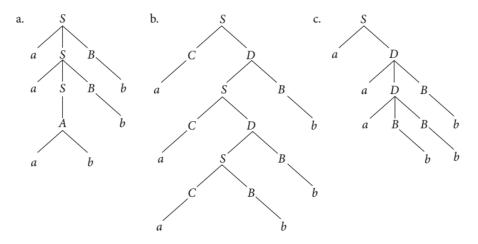


Figure 4: Derivations trees for word a^3b^3 . **a.** derivation in grammar G, **b.** derivation in grammar G' (Chomsky normal-form), **c.1** derivation in grammar G'' (Greibach normal-form)

Grammar example: mirror language

Grammar

Let
$$G = (V_N, V_T, P, S)$$
, where

•
$$V_N = \{S\}$$

•
$$V_T = \{a, b\}$$

•
$$P = \{S \rightarrow aSa, S \rightarrow bSb, S \rightarrow aa, S \rightarrow bb\}$$

Derivations

$$\mathsf{S} \implies \mathsf{aSa} \implies \mathsf{abSba} \implies \mathsf{abbSbbs} \implies \mathsf{abbaSabba} \implies \dots$$

Language

 $L(G) = \{ww^R\}$, where w^R represents reflection of w, and $|w| \ge 1$. This language L(G) is called a *mirror language*.

Example sentences

aa, bb, aaaa, abba, baab, bbbb, abaaba, baaaab, abbbbba, babbab, aaaaaa...

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Grammar example

Grammar

Let
$$G = (V_N, V_T, P, S)$$
, where

•
$$V_N = \{S, E, F\}$$

•
$$V_T = \{a, b, c, d\}$$

•
$$P = \{S \rightarrow ESF, S \rightarrow EF, E \rightarrow ab, F \rightarrow cd\}$$

Derivations

$$S \implies ESF \implies EESFF \implies EEESFFF \implies E^{n-1}SF^{n-1} \implies E^nF^n$$

Language

$$L(G) = \{(ab)^n (cd)^n\}, \text{ where } n \geq 1.$$

Example sentences

 $abcd, ababcdcd, abababcdcdcd, ababababcdcdcdcd, \ldots$

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Grammar example

Grammar

- Let $G = (V_N, V_T, P, S)$, where
 - $V_N = \{S, E, F\}$
 - $V_T = \{a, b, c, d\}$
 - $P = \{S \rightarrow ESF, S \rightarrow abcd, Ea \rightarrow aE, dF \rightarrow Fd, Eb \rightarrow abb, cF \rightarrow ccd\}$

Derivations

 $\mathsf{S} \Rightarrow \mathsf{ESF} \Rightarrow \mathsf{EabcdF} \Rightarrow \mathsf{aEbcdF} \Rightarrow \mathsf{aEbcFd} \Rightarrow \mathsf{aabbcFd} \Rightarrow \mathsf{aabbccdd}$

Language

$$L(G) = \{a^n b^n c^n d^n\}, \text{ where } n \geq 2.$$

Sentences

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A few common tasks:

- Check language correctness
- Generate language from grammar
- Generate grammar for language
- Classify grammar (according to the Chomsky hierarchy)

Backus-Naur form (BNF)

Notation technique for *context-free grammars*. Frequently used to describe syntax of *programming languages*, *document formats* etc.

Syntax

<term> ::= __expression__

- <term> is a nonterminal
- __expression__ is a sequence of one or more terminal and/or nonterminal symbols separated by vertical line |
- Terminal symbols: a, b, c, A, 0, 1, 2 etc.
- Nonterminal symbols: <digit>, <postal-code> etc.

Backus-Naur form (BNF)

Meta-symbols

- ::= production rule definition
- | rule alternative
- <> nonterminals
- "" literal
- < EOL > End Of Line

Examples

```
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<postal-code> ::= <digit> <digit> <digit> <digit> <digit> <digit>
```

BNF example : Palindrome

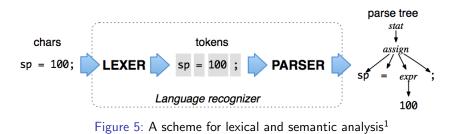
Palindrome grammar

<letter></letter>	::= a b c y z	
<palindrome></palindrome>	::= <letter> </letter>	
<palindrome></palindrome>	::= a <palindrome> a b <palindrome> b</palindrome></palindrome>	b
	c <palindrome> c d <palindrome> c</palindrome></palindrome>	f
	e <palindrome> e </palindrome>	
	z <palindrome> z</palindrome>	z

Results	
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pop	
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Postal address grammar

```
<postal-address> ::= <name-part> <street-address> <zip-part>
<name-part> ::= <first-name> <last-name> <EOL>
<street-address> ::= <number> <street-name> <apt-num> <EOL>
<zip-part> ::= <postal-code> <town-name> <EOL>
<apt-num> ::= <number> | ""
```



- Lexical analysis converts a sequences of characters into a sequence of tokens (strings with meanings)
- Semantic analysis matches a sequence of tokens into some language's construct

¹From ANTLR4 on-line documentation

ANTLR v4

Parser generator

Generate lexer and parser for given grammar.

ANTLR

A parser generator which allows to:

- Perform lexical analysis (lexer)
- Perform semantic analysis (parser)
- Code transformation
- Perform code evaluation

Usages

- Twitter search queries are parsed using ANTLR
- Lex Machina^a extracts informations from legal texts using ANTLR

^alexmachina.com

ANTLR syntax (I)

Grammar structure

```
grammar ANY_NAME;
options {...}
import ...;
tokens {...}
channels {...}
@actionName {...}
// lexer rules
LEXER_RULE1
LEXER_RULE2
// parser rules
parser_rule1
parser_rule2
```

Grammar properties

- Each section can be specified in any order
- Only one definition for sections: *options*, *imports*, *tokens*
- The header and at least one rule are mandatory

Reserved keywords

import, fragment, lexer, parser, grammar, returns, locals, throws, catch, finally, mode, options, tokens

Grammar file

The file name with grammar ANY_NAME must be called ANY_NAME.g4

ANTLR syntax (II)

Syntax	Description				
X	Match token, rule or subrule x				
xyz	Match a sequence of elements				
()	Sub-rule with multiple alternatives				
x?	Match x or skip it				
<i>X</i> *	Match x zero or more times				
x+	Match x one or more times				
<i>r</i> :	Define rule <i>r</i>				
r : ()	Define rule r with multiple alternatives				

Simple grammar (Hello.g4)

```
// define a grammar called Hello
grammar Hello;
// match lower-case identifiers
ID : [a-z]+;
// skip spaces, tabs, newlines, \r (Windows)
WS : [ \t\r\n]+ -> skip;
// match keyword hello followed by an identifier
r : 'hello' ID;
```

Nested arrays grammar (ArrayInit.g4)

```
grammar ArrayInit;
```

```
// matches at least one comma-separated value between {...}
init : '{' value (',' value)* '}';
// A value can be either a nested array or an integer (INT)
value : init | INT;
// define token INT as one or more digits
INT : [0-9]+;
WS : [ \t\r\n]+ -> skip;
```

ANTLR patterns

Pattern name	Examples
Sequence	'[' INT+ ']'
Sequence with terminator	(statement ';')*
Sequence with separator	(expr (',' expr)*)?
Choice	type : 'int' 'float'
Token dependency	ID '[' expr ']'
Nested phrase	expr : '(' expr ')' ID

Parser

```
import org.antlr.v4.runtime.*;
import org.antlr.v4.runtime.tree.*;
public class Test {
    public static void main(String[] args) throws Exception {
        // create a CharStream that reads from standard input
        ANTLRInputStream input = new ANTLRInputStream(System.in);
        // create a lexer that feeds off of input
        CharStream ArrayInitLexer lexer = new ArrayInitLexer(input);
        // create a buffer of tokens pulled from the lexer
        CommonTokenStream tokens = new CommonTokenStream(lexer);
        // create a parser that feeds off the tokens buffer
        ArrayInitParser parser = new ArrayInitParser(tokens);
        ParseTree tree = parser.init();
        System.out.println(tree.toStringTree(parser));
    }
}
```

Calculator

```
grammar Expr;
prog: stat+;
stat: expr NEWLINE
    | ID '=' expr NEWLINE
    NEWLINE;
expr: expr ('*'|'/') expr
    | expr ('+'|'-') expr
    INT
    I ID
     '(' expr ')';
ID : [a-zA-Z]+;
INT : [0-9]+;
// return newlines to parser (is end-statement signal)
NEWLINE: '\r'? '\n';
WS : [ \t] + ->  skip;
```