# TD

# THEORIE DU SIGNAL

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## I. FOURIER TRANSFORM

1-Donner la Transformée de Fourier de :

$$s(t) = \cos 2\pi \frac{t}{T} . \mathrm{Re}ct \frac{(t - T_0/2)}{[T_0]}; \quad T_0 \rangle T$$

**2-** Please give the Fourier Transform of the signal (generalised function or tempered distribution)

$$s(t) = t^2$$

NB : We recall that :

$$\mathscr{T}\left[\left((-j2\pi t)^n g(t)\right)\right] = \hat{g}^{(n)}(\nu) \text{ and } \mathscr{T}\left[g(t) = 1\right] = \hat{g}(\upsilon) = \delta$$

3-Montrer que pour 2 fonctions $f$ *et* $g$ qui possèdent une transformée de Fourier

alors $\langle f, g \rangle = \langle \hat{f}, \hat{g} \rangle$

## II. DISCRETE FOURIER TRANSFORM (DFT)

*if* $x_n = u(n) - u(n - N)$ *with* $N = 4$ and *let give* $\hat{x}_k$ *and sketch it*

With $\hat{x}_k$ the Discrete Fourier Transform of $x_n$

NB : we recall that $\hat{x}_k = \sum_{n=0}^{N-1} x_n e^{-2j\pi \frac{nk}{N}}$

## III. PROBABILITY

**A-** $X$ is a random variable with a probability density function (pdf)

$$f_X(x) = \begin{cases} kx & 0 \langle x \langle 1 \quad k = const. \\ 0 & otherwise \end{cases}$$

1- Find $k$ and sketch $f_X(x)$

2- Find and sketch the cumulative distribution function (cdf) $F_X(x)$

3- Find $P\left\{\frac{1}{4} \langle X \le 2\right\}$

4- Give the mean $m = E(X)$ and the variance $\sigma_X^2$

NB : we recall that $F_X(x) = P\{X \le x\}$

**B** – Consider a random process defined by

$$X(t) = Y\cos(\omega t + \Phi)$$

Where $Y$ and $\Phi$ are independent random variables and are uniformly distributed over $(-A + A)$ and $(-\pi + \pi)$ respectively.

1-Find the mean of $X(t)$

2-Find the expression of the autocorrelation $R_{XX}(t_1, t_2)$

3- Is $X(t)$ WSS (Wide Stationarity Sense) ?

**C** – Suppose that a random process $X(t)$ is WSS with autocorrelation

$$R_{XX}(\tau) = \exp - \frac{|\tau|}{2} \quad : (t_1 - t_2 = \tau)$$

1- Find the second moment of the random variable $X(5)$ which is $E\left[X^2(5)\right]$

2- Find the second moment of the random variable $X(5) - X(3)$ which is

$E\left[X(5) - X(3)\right]^2$

**D-** We considere the continuous and complex process $X$ defined by :

$$X(t,\xi) = \sum_{k=1}^{n} A_k \exp\left[j\omega t + j\theta_k(\xi)\right]$$

Where $\theta_k$ are independent random variables $\forall k = 1, ...., n$ and uniformaly distributed on $[0, 2\pi]$ and $A_k \in \mathbb{R}$

1-Give the mean value of $X$

2-Give its autocorrelation function

3-Give its variance

4-Is $X$ a WSS ?